جامعة بنـها	الفرقة: الثانية تربية(رياضة)
كلية التربية	د./ صلاح عيد ابراهيم حمزة
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1. Using the dimensional analysis, derive an expression for the time period of oscillation of a simple pendulum. Assuming that the time period depends on (i) mass, (ii) length and (iii) acceleration due to gravity

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Assume that t, m, ℓ and g are related through the equation:

$$t \propto m^x \ell^y g^z$$

$$t = k m^x \ell^y g^z$$

By using the dimensional method

$$T = M^x L^y (L T^{-2})^z$$

$$M^0 L^0 T^1 = M^x L^{y+z} T^{-2z}$$

Comparison the powers of M, L and T on both sides

$$x = 0, \quad y + z = 0, \quad -2z = 1$$

Solving the three equations,

$$x = 0, \quad y = \tfrac{1}{2}, \quad z = -\tfrac{1}{2}$$

$$\therefore \; t = k\sqrt{\frac{\ell}{g}}$$

2(a) Derive an expression for the moment of inertia of a cylinder about its axis.

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Consider a uniform circular cylinder of length L, mass m and radius R rotating about an axis passing through its axis, as in Fig. (8). Consider an element of length L, radius r and radial thickness dr . Then

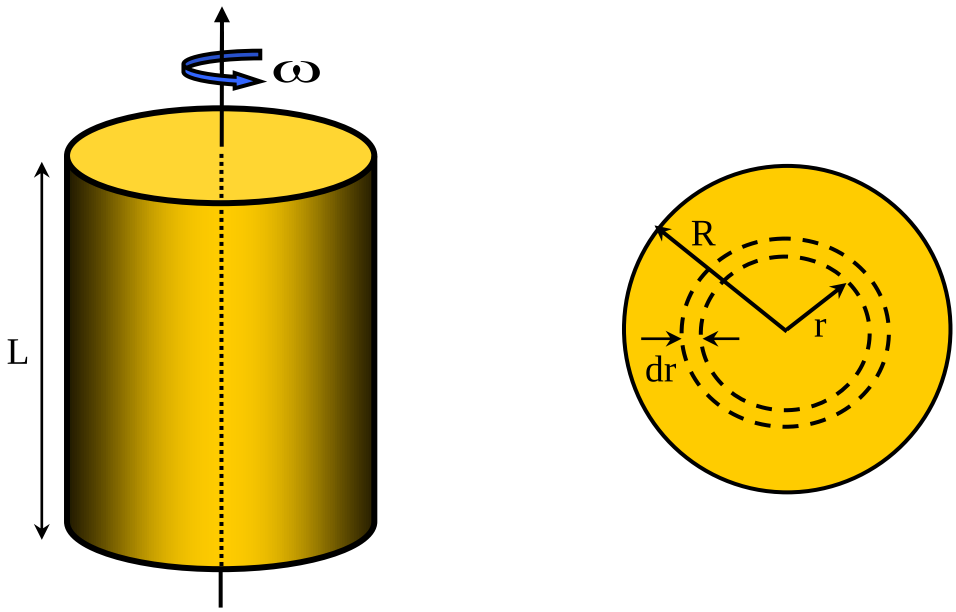

Figure (8)

1. Volume of the cylinder $V = \pi R^2 L$
2. Total mass of the cylinder $m = \rho \pi R^2 L$
3. Volume element $V = \pi r^2 L \Rightarrow dV = 2\pi r\, dr\, L$
4. Moment of inertia of the cylinder $I = \int \rho r^2\, dV$

$$I = \int_0^R \rho r^2 \times 2\pi r L \; dr$$

$$I = 2\pi\rho L \int_0^R r^3 \; dr \quad = 2\pi\rho L \times \tfrac{1}{4} r^4 \Big|_0^R$$

$$= \tfrac{1}{2}\pi\rho L R^4 \quad , \qquad m = \pi\rho R^2 L$$

$$I = \tfrac{1}{2} m R^2$$

2(b) Show that the time period of oscillations of loaded spring is $T = 2\pi\sqrt{m/k}$

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As a model for simple harmonic motion, consider a block of mass m attached to the end of a spring, with the block free to move on a horizontal, frictionless surface, as in Fig. (4). When is neither stretched, the block is at the position called the equilibrium position of the system, which identify as $x = 0$.

We can understand the motion in Fig. (4) by recalling that when the block is displaced to the position x, the spring exerts on the block a force that is proportional to the position and given by Hook's law:

$$F = -k x$$

We call this a ***restoring force*** because it is a always directed toward the equilibrium position and therefore *opposite* the displacement from equilibrium. That is, when the block is displaced to the right of $x = 0$ in Fig. (4a), then the position is positive and the restoring force is directed to the left. When the block is displaced to the left of $x = 0$, then the position is negative and the restoring force is directed to the right.

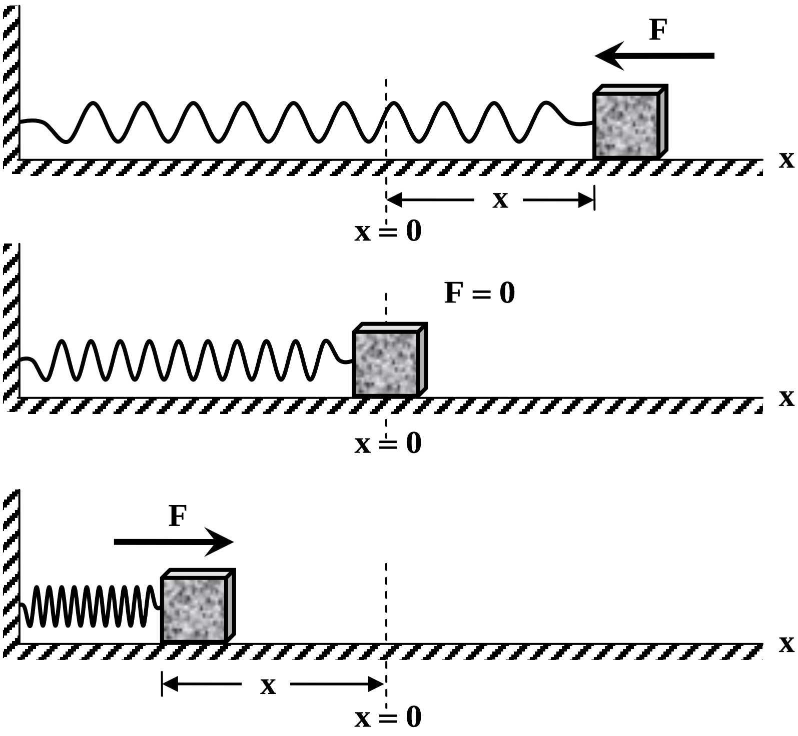

Figure (4)

Applying Newton's second law to the motion of the block, we obtain:

$$F = -k\,x$$

$$m\,\ddot{x} = -k\,x$$

$$\ddot{x} = -\frac{k}{m}x$$

This equation is similar to the equation of simple harmonic motion

$$\ddot{y} = -\omega^2 y$$

Comparing the last two equations:

$$\omega = \sqrt{\frac{k}{m}} \quad \Rightarrow \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

3(a) Derive an expression for the wave equation.

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The equation of any wave is the solution of a different equation called the wave equation. To establish the form of the wave equation, let us compare the second partial derivatives with respect to the coordinates and time of the function describing a plane wave. In the last chapter we see that, the equation describe a wave propagate in the negative x-direction is given by:

$$y = A\sin(kx + \omega t) \qquad (1)$$

Differentiating this equation twice with respect to coordinate, x, we get

$$\frac{dy}{dx} = kA\cos(kx - \omega t)$$

$$\frac{d^2y}{dx^2} = -k^2 A\sin(kx - \omega t)$$

$$\frac{d^2y}{dx^2} = -k^2 y \qquad (2)$$

Differentiating Eq. (1) twice with respect to time, t, we get

$$\frac{dy}{dt} = -\omega A\cos(kx - \omega t)$$

$$\frac{d^2y}{dt^2} = -\omega^2 A\sin(kx - \omega t)$$

$$\frac{d^2y}{dt^2} = -\omega^2 y$$

Substituting about $\omega = k\upsilon$

$$\frac{d^2y}{dt^2} = -k^2\upsilon^2 y \qquad (3)$$

A comparison between Eq. (2) and (3) gives:

$$\frac{d^2y}{dt^2} = \upsilon^2 \frac{dy^2}{dx^2} \tag{4}$$

This is exactly the wave equation. Any function satisfying an equation of the form of Eq. (4) describes a wave; the square root of the quantity that is the coefficient of $\frac{d^2y}{dx^2}$ gives the phase velocity of this wave.

3 (b) Prove that the function $y = \sin x \cos(\upsilon t)$ represents a solution of the wave equation

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If the given function y represents a solution of the wave equation, it must be satisfy it. So, differentiate y twice with respect to x and t, then substitute in the wave equation

$$y = \sin x \cos(\upsilon t)$$

$$\frac{dy}{dx} = \cos x \cos(\upsilon t)$$

$$\frac{d^2y}{dx^2} = -\sin x \cos(\upsilon t) = -y$$

$$\frac{dy}{dt} = -\upsilon \sin x \sin(\upsilon t)$$

$$\frac{d^2y}{dt^2} = -\upsilon^2 \sin x \cos(\upsilon t) = -\upsilon^2 y$$

$$\frac{d^2y}{dt^2} = \upsilon^2 \frac{d^2y}{dx^2}$$

So the equation $y = \sin x \cos(\upsilon t)$ represent a wave equation.